

When Do We Know That We Need To Use Parts?

When the integral is a **product** of two completely **unrelated functions** (and we can't use reverse chain rule or substitution) such as:

- \int **Power \times Trig** e.g. $\int x \sin x \, dx$ or $\int x^2 \sin x \, dx$
- \int **Power \times Exponential** e.g. $\int x e^x \, dx$ or $\int x^2 e^x \, dx$
- \int **Easy power \times Harder power** e.g. $\int 5x\sqrt{2-x} \, dx$
- \int **Power \times Ln** e.g. $\int x \ln x \, dx$ or $\int x^2 \ln x \, dx$ or $\int \frac{1}{x} \ln x \, dx$
- \int **Exponential \times Trig** e.g. $\int e^x \sin x \, dx$ or $\int e^x \cos x \, dx$

There are two harder types (natural logarithm and inverse trig) that aren't a product and it is not obvious to use parts. These types need to be remembered as special cases and turned/forced into a product in order to use parts:

- $\int \ln x = \int 1 \times \ln x$
- $\int \sin^{-1} x = \int 1 \times \sin^{-1} x$
- $\int \cos^{-1} x = \int 1 \times \cos^{-1} x$
- $\int \tan^{-1} x = \int 1 \times \tan^{-1} x$

A good rule of thumb to follow if you want to know whether to use parts would be to try a substitution first (so make sure you're good at substitution before learning parts), and then if the resulting doesn't become easier to integrate once simplified, try integration by parts. A substitution is mostly used (but certainly not always) whenever we have a **function f containing a function** (we'll call this g) multiplied by the **derivative of g** . That is, if you have the form $\int g'(x)f(g(x))dx$. For example,

$$\int x e^{x^2} dx$$

$$f(g(x)) = e^{x^2} \text{ so } f = e^x, g = x^2$$

The **derivative** of x^2 is $2x$ and we have the correct x term. It doesn't matter if the constant 2 in front is different. In this case we would use substitution (or reverse chain rule if you're good at integration) instead of parts. **It is worth mentioning that parts would still work, it would just take longer! So, lots of integrals that can be done by substitution can be done with parts instead, but the converse is not true.**

The best strategy is to try to "play it out" in your mind i.e. whether substitution or parts will be better and try to see which one will work better. The only way to get better at these sorts of integrals is to practice large sets of each type. Then, you start to think "Oh—this looks like a substitution!" or, "maybe by-parts is better for this." **Practise, practise, practise is really the best way to get better at recognizing whether to use substitution or parts and is key to becoming good at parts.**

How Do We Apply Parts?

We say our original integral has the form/looks like $\int u \frac{dv}{dx} dx$ or $\int \frac{dv}{dx} u dx$

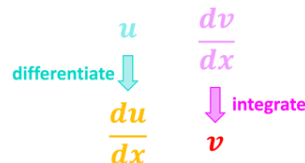
Step 1:

Call one of the functions u and the other function $\frac{dv}{dx}$. It matters which we call u and which we call $\frac{dv}{dx}$!

See **and the table on the next page for how to choose which function is u and which is $\frac{dv}{dx}$

Step 2:

Find $\frac{du}{dx}$ by differentiating u and v by integrating $\frac{dv}{dx}$

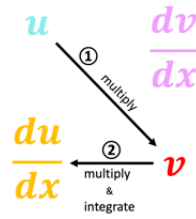


Notice how for both functions we want to end up with the original function and its corresponding derivative which leads us to step 3.

Step 3:

Plug into the formula $uv - \int v \frac{du}{dx} dx$

It can help to remember the formula as ① - ②



Step 4:

Simplify the integral part $\int v \frac{du}{dx} dx$ as much as possible before integrating. You take out constants (that are multiplied or divided) to make the integration easier.

****How we know which function to call u and which do we call $\frac{dv}{dx}$ ****

We need to call u the function that we can differentiate and $\frac{dv}{dx}$ the function which we can EASILY integrate. Recall that our functions are either algebraic functions i.e. x^n , trig functions, logarithmic functions, exponentials or inverse trig functions. There are many 3 ways you can decide which function to use for u and which function to use for $\frac{dv}{dx}$:

<p>Way 1: Best Method For Beginners Use the acronym LIATE</p>	<p>Way 2: <u>Memorise</u> the following templates for the different types of functions</p>	<p>Way 3: Think ahead and play it out what will happen in mind</p>
<p>Consider all the possible types of functions that can come up:</p> <p>Logarithms $\ln x, \log x$</p> <p>Inverse Trig $\sin^{-1} x, \cos^{-1} x, \tan^{-1} x$</p> <p>Algebra constants, x, x^2, \dots</p> <p>Trigonometry $\sin x, \cos x, \tan x$</p> <p>Exponentials e^x</p> <p>Locate the two functions given inside in the integral. Whichever of the 2 functions that comes first on the list above is always called u.</p> <p>For example:</p> $\int x^3 \ln x dx$ <p>x^3 is the Algebra function $\ln x$ is the Logarithmic function The Logarithmic function (L) comes before Algebra (A) on the list above so we call logarithmic function u and the algebraic function $\frac{dv}{dx}$</p>	<p>blue functions below are u and pink functions below are $\frac{dv}{dx}$</p> <p>The types we encounter are:</p> <ul style="list-style-type: none"> $\int (\text{Power})(\text{Trig})$ eg. $\int (x^2)(\sin x)$ $\int (\text{Power})(\text{Exponential})$ eg. $\int (x^2)(e^x)$ $\int (\text{Easy power})(\text{Harder power})$ eg. $\int x \sqrt{x-4}$ eg. $\int (x+2)(x+5)^5$ $\int (\text{Easy power})(\ln)$ eg. $\int (x^2)(\ln 2x)$ $\int (\text{Exp})(\text{Trig})$ or $\int (\text{Exp})(\text{Trig})$ (the order you pick the functions actually doesn't matter for this type) <p>Two harder types that aren't a product which need to be turned into a product:</p> $\int \ln = \int 1 \times \ln$ $\int \text{Inverse trig} = \int 1 \times \text{Inverse trig}$	<p>This way comes naturally once you have more experience.</p> <p>We decide which function to call "u" first. This is the function that will become an "easier" function when differentiated. For example</p> $\int x^3 \sin x dx$ <p>x^3 becomes $3x^2$ when differentiated i.e. when we differentiate "u" we "kill" powers. $\sin x$ becomes $\cos x$ when differentiated. This is not simpler. We always look to make $\frac{du}{dx}$ simpler than "u", since this will make the resulting latter integral part of the formula $\int v \frac{du}{dx} dx$ easier to integrate.</p> <p>The other function is then automatically $\frac{dv}{dx}$. Remember the function that you call $\frac{dv}{dx}$ is the function you must be able to integrate, so never call a function $\frac{dv}{dx}$ that you are unable integrate.</p>